SECOND EUROPEAN CONFERENCE ON EARTHQUAKE ENGINEERING AND SEISMOLOGY, ISTANBUL AUG. 25-29, 2014

SIMPLIFIED SEISMIC VULNERABILITY ASSESSMENT PROCEDURE FOR REINFORCED CONCRETE BRIDGES

Alessandro ZANARDI[1], Alberto PAVESE[2], Simone PELOSO[3] and Cristiano RENNA[4]

ABSTRACT

In this paper a simplified assessment procedure for multi-span, reinforced concrete (R.C.) bridges will be described and a numerical example will be presented. The proposed procedure is similar to the V.C. Method developed by Dolce et al. (2005) for reinforced concrete frames, conveniently modified in order to be applied to this different kind of structures.
The procedure determines the capacity curve of each pier, which is axial load and reinforcement dependent, taking into account three possible collapse mechanisms: shear, sliding and flexure.
The simplified collapse estimate of the numerical example has been compared to the one evaluated by means of time history analyses, performed on a nonlinear model which has been validated on the results of pseudo-static tests done on two 1:2 scaled pier specimens.

Keywords: simplified analyses; fast assessment procedure; reinforced concrete bridges; collapse prediction.

INTRODUCTION

The fast seismic vulnerability assessment of structures is particularly important during post-earthquake crises. Nowadays, after a seismic event, structures are visually inspected and their safety is determined observing the accumulated damage. This is not an universal criterion and it could lead to different judgments depending on the experience of the evaluation team.

The paper proposes an alternative fast assessment method for reinforced concrete (R.C.) bridges, based on the V.C. Method developed by Dolce et al. (2005), which leads to a more objective assessment. The authors, in a previous research about R.C. frame assessment (Peloso et al., 2012), analysed the main parameters affecting the results of the simplified method and found that these parameters could be better determined by means of experimental tests.

In this article the original VC Method, developed for R.C. frames, will be modified to be applied to R.C. bridges.

The dynamic response of bridges is clearly different from the response of frames: usually bridges have piers with different heights, decks with various possible construction technologies and connections. Important modifications to the original method need to be considered to account for these key factors.

[1] PhD Student, UME School, Pavia, alessandro.zanardi@umeschool.it
[2] Professor, University of Pavia, Pavia, a.pavese@unipv.it
[3] PhD, Eucentre Foundation, Pavia, simone.peloso@eucentre.it
[4] M.Eng. Student, University of Pavia, Pavia, cristiano.renna01@universitadipavia.it

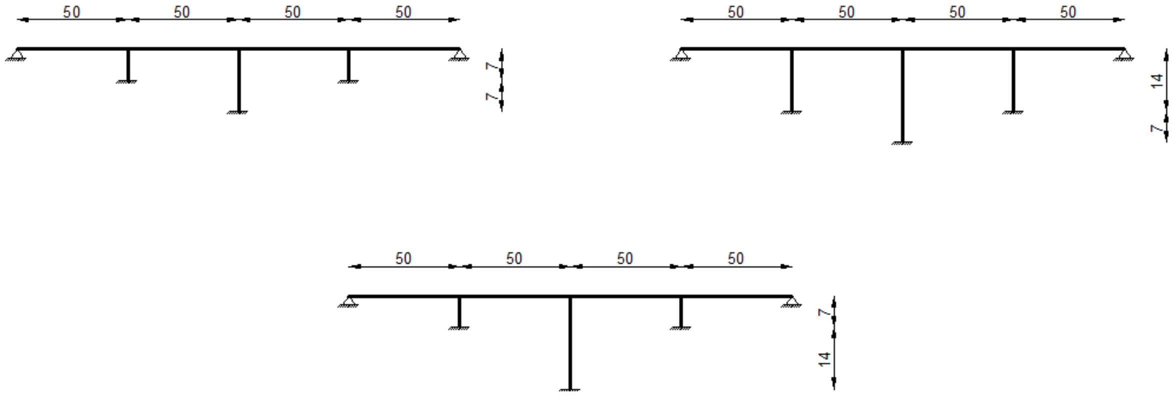

Figure 1. Some of the bridges configurations investigated with modal analyses (measures in metres)

The present research focuses on R.C. bridges with symmetric configuration, 4 constant length bays, pier heights ranging from 7 m to 21 m, tallest pier at the centre of the bridge and continuous deck with constant section (see Fig. 1). Next Fig. 2 shows the finite element model (FEM) of the bridge described in the example.

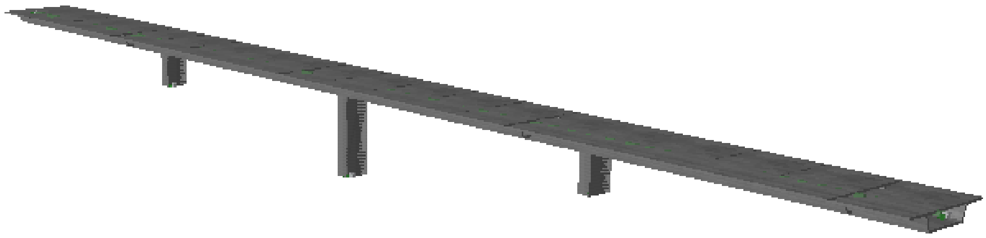

Figure 2. Model of the analysed bridge in the example

One of the key parameter affecting the simplified estimate of the collapse parameters is the first structural natural period. Several modal analyses have been performed on eight different bridges and it has been observed that the first fundamental frequency strongly depends on the deck stiffness.

An ad hoc developed index, which analyses the deck influence on the whole modal behaviour, will be presented and a simplified relationship, which could be employed for the estimation of the first period of the bridge when a dynamic in-situ identification or a modal analysis is not available, will be proposed.

The simplified procedure continues estimating the lateral shear capacity of each pier, a displacement horizontal profile and, through a code elastic acceleration spectrum, the collapse peak ground acceleration. A set of simple coefficients is then used to improve the collapse estimate (PGA):

$$PGA = \frac{s_d \cdot \alpha_{DUT}}{\alpha_{AD} \cdot \alpha_{DS}} \qquad (1)$$

The coefficients in Eq. 1 are accounting for the pier strengths (s_d), the mean force reduction factor (α_{DUT}), the dissipation capacities of the structure (α_{DS}) and the spectral amplification factor (α_{AD}).

The simplified method has been applied to the bridge with the following pier heights: 7 for the side piers and 14 m for the central one. The collapse peak ground acceleration estimated with the simplified method has been compared to the results obtained from dynamic nonlinear analyses carried out with a nonlinear numerical model, calibrated on experimental tests.

The results of the experimental tests and the numerical simulations will be presented and discussed.

THE PROPOSED METHOD FOR REINFORCED CONCRETE BRIDGES

A simplified procedure, based on the V.C. Method developed by Dolce et al. (2005), has been implemented to assess the seismic vulnerability of R.C. bridges with a continuous deck and a symmetric configuration, having the tallest pier at the centre of the bridge. For this kind of structures the deck is not significantly affected by the seismic load, thus is assumed elastic.

The input data required are: the geometric element dimensions, the longitudinal and transversal reinforcement in each pier and the materials (concrete and steel) mechanical characteristics.

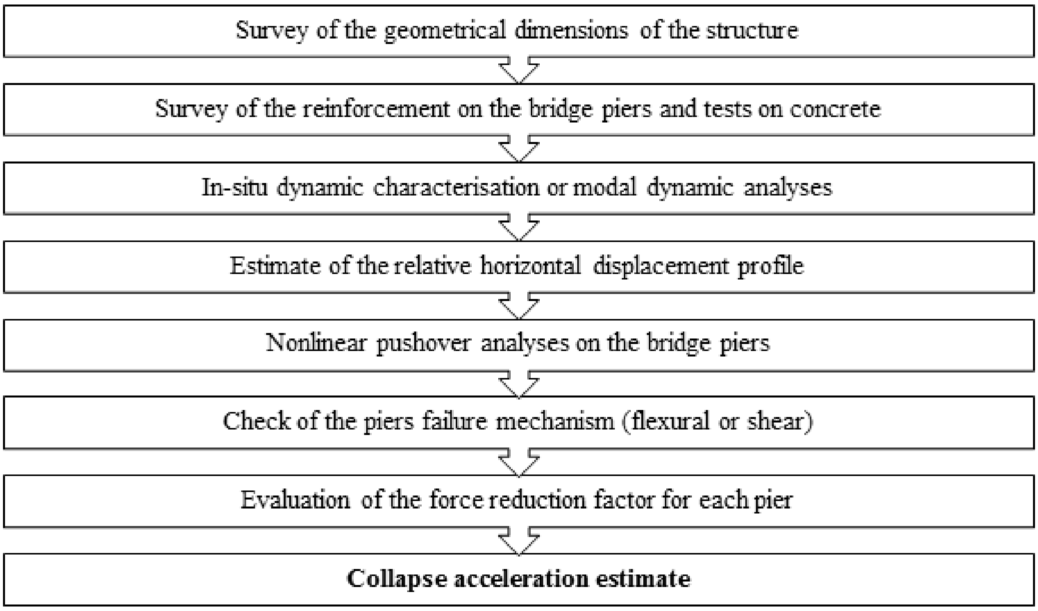

Figure 3. Flowchart of the proposed procedure

Fig. 3 summarizes the method procedure: after the survey of the geometrical dimensions and the tests on materials, the seismic axial load on the piers, according to the Eurocode 8 suggestion, is evaluated. Then the displacement capacities of each pier are assessed by means of pushover nonlinear analysis performed on each single pier, where the horizontal load in each vertical element is increased until its flexural collapse.

The nonlinear analysis is performed with fibre displacement-based elements, having an element length in the vertical direction equal to about the 2% of the higher pier height. The concrete nonlinear material model is chosen to reproduce the mechanical behaviour observed from the non-destructive tests on the structure while the steel properties are those available in literature related to the material used at the time of construction, with the safe assumption of a 0.04 ultimate steel strain.

A shear elastic spring (K_t) is modelled at the top of pier, with a stiffness evaluated as in Eq. 2 where G is the shear modulus of concrete, A the area of the cross-section, h the pier height and χ the shear correction factor, to account for the shear pier deformation. The elements response is also including P-Delta effects, which could affect significantly the response in tall piers.

$$K_t = \frac{G \cdot A \cdot \chi}{h} \tag{2}$$

The ultimate displacement (Δ_u) at the top of each pier is limited to the value expressed by Eq. 3:

$$\Delta_u = \theta_u \cdot H_P \tag{3}$$

$$\theta_y = \phi_y \cdot \frac{l_v}{3} \tag{4}$$

$$\theta_u = \frac{1}{\gamma_{el}}\left[\theta_y + (\phi_u - \phi_y)\cdot L_P \cdot \left(1 - \frac{L_P}{2\cdot l_v}\right)\right] \tag{5}$$

where γ_{el} is a safety factor equal to 1.5 as suggested by the Italian Code, the length of the plastic hinge L_P is equal to the 10% of the pier height, l_v is the shear length (M/V), ϕ_y and ϕ_u are the yielding and ultimate curvature of the section respectively evaluated for the seismic axial load, θ_y is the yielding chord rotation evaluated accordingly to Eq. 4, H_P is the pier height and θ_u is the ultimate chord rotation estimated using Eq. 5.

Depending on the acting axial load, the shape of the section and the steel reinforcement, the pier collapse could be reached with a ductile (flexural) or a brittle (shear) mechanism. If a ductile mechanism is ensured, the structural element is able to dissipate energy proportionally to its ductility, otherwise the response of the vertical element is considered elastic up to the shear failure.

The failure mechanism is investigated comparing, as shown in Fig. 4, the capacity curve from the pushover analysis and the shear envelope, evaluated according to Priestley et al. (2007).

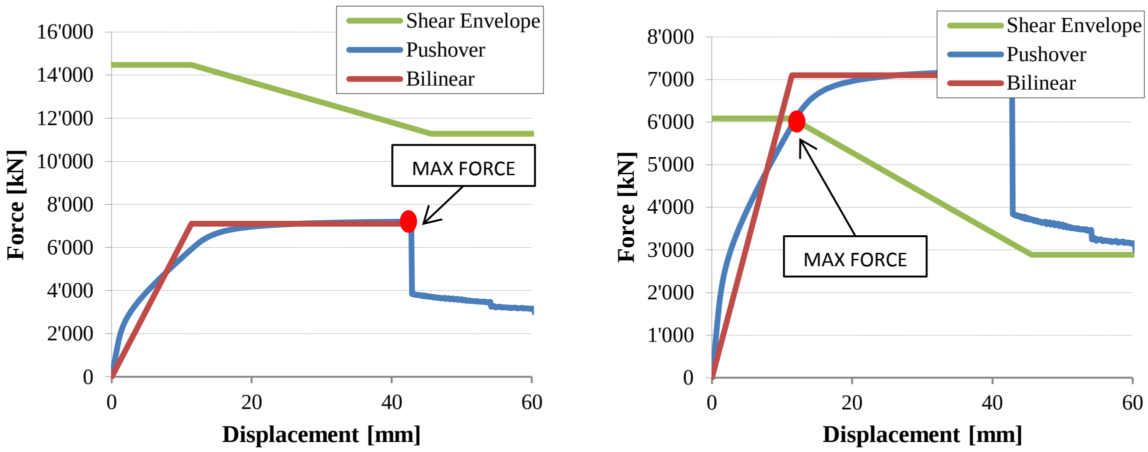

Figure 4. Comparison between a ductile (left) and brittle (right) mechanism

In more detail, the shear capacity of each pier is determined adding the shear strength provided by the concrete mechanism (V_C), the truss reinforcement truss mechanism (V_S) and the axial force one (V_P), as in Eq.6:

$$V_D = V_C + V_S + V_P \tag{6}$$

$$V_C = k\cdot\sqrt{f_c}\cdot 0.8\cdot A_C \tag{7}$$

$$V_S = \frac{A_V\cdot f_y\cdot D'\cdot\cot\theta}{s} \tag{8}$$

$$V_P = P\cdot\tan\alpha \tag{9}$$

where k is the concrete shear mechanism strength, ductility dependent, f_c is the concrete compressive strength, A_C the area of the concrete section, A_V the effective area of hoops, f_y the steel yielding stress, D' is the core dimension, s the spacing between stirrups, θ the angle of the flexure-shear crack which can generally be assumed to be equal to 30°, P is the seismic axial load on the pier and α is the inclination of the strut involving the axial load. Additionally the pier is verified against sliding shear failure, with the Italian Code model, see Eq. 10:

$$V_{Sl} = V_{dd} + V_{fd} \tag{10}$$

$$V_{dd} = \min\begin{cases} 1.3 \cdot \sum A_{sj} \cdot \sqrt{f_c \cdot f_y} \\ 0.25 \cdot f_y \cdot \sum A_{sj} \end{cases} \tag{11}$$

$$V_{fd} = \min\begin{cases} \mu_f \cdot \left[\left(\sum A_{sj} \cdot f_y + N\right) \cdot \xi + M / z\right] \\ 0.5 \cdot \eta \cdot \xi \cdot f_c \cdot l \cdot b \end{cases} \tag{12}$$

where A_{sj} is the area of the longitudinal reinforcement, f_c and f_y are the concrete compressive strength and the steel yielding stress, ξ is the ratio of the neutral axis to the section depth, η could be evaluated with Eq. 13, μ_f is the concrete friction factor equal to 0.6, N is the seismic axial load, l and b are the section dimensions.

$$\eta = 0.6 \cdot \left(1 - \frac{f_c}{250}\right) \tag{13}$$

In order to understand which pier is failing and which is the actual failure mechanism (shear or flexure), a horizontal displacement profile of the deck must be assumed. A simplified displacement profile has been implemented in the procedure: the top displacement of each pier (Δ_{Pi}) is equal to the pier height (H_{Pi}), normalized to the highest pier height, multiplied by the a scale factor (α), as in Eq. 14.

$$\Delta_{Pi} = \left(\frac{H_{Pi}}{\max(H_{Pi})}\right) \cdot \alpha \tag{14}$$

The horizontal displacement profile is giving the normalized top displacement of each pier and should be scaled up to the failure of the first vertical element. This allows to define the minimum scale factor (α) and all the forces in the piers, estimated through the respective capacity curves. Furthermore the actual failure mechanism is identified: if the pier shear strength is higher than the force experienced in each element for the assigned displacement, a flexural mechanism is ensured otherwise a brittle failure is expected.

Knowing the collapse displacement profile, the ductility of each pier is then evaluated comparing the estimated displacement (Δ_{Pi}) with the yielding displacement of each element ($\Delta_{i,y}$), as in Eq. 15:

$$\mu_i = \frac{\Delta_{Pi}}{\Delta_{i,y}} \geq 1 \tag{15}$$

A force reduction factor (q_i), based on the equal displacement assumption, is evaluated for each pier with a ductile mechanism, assuming it equal to ductility of the vertical element. If a brittle failure is expected in the pier, then the force reduction factor is assumed equal to 1. Finally the collapse peak ground acceleration is evaluated with the Eq. 16. The total base shear is equal to the sum of the maximum pier forces (F_i), each multiplied by their reduction factor (q_i). Three different situations could occur: 1) if the pier fails with a flexural mechanism, then F_i is equal to yielding force from the elastoplastic bilinearization of the capacity curve and q_i is equal to the ductility; 2) if the pier fails with a shear mechanism, then F_i is equal to the pier shear strength and q_i is equal to 1; 3) if the pier is still elastic, then F_i is equal to the force observed in its capacity curve for the assigned pier displacement and q_i is equal to 1.

$$PGA = \frac{\sum F_i \cdot q_i}{9.81 \cdot \alpha_{DS} \cdot M_{eff} \cdot \alpha_{AD}} \tag{16}$$

A set of simple coefficients are introduced in Eq. 16 to improve the collapse estimate: α_{DS} is the dissipation factor evaluated according to Eq. 17 where ξ is the assumed damping percentage, α_{AD} is the spectral amplification factor evaluated for the first fundamental frequency in the weaker direction and M_{eff} is the effective modal mass, computed evaluating only the sum of the pertinence mass of each pier, as shown in Fig. 5.

$$\alpha_{DS} = \sqrt{\frac{10}{5+\xi}} \tag{17}$$

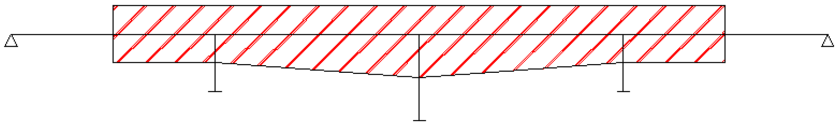

Figure 5. Effective modal mass

PERIOD RELATION OF THE ANALYSED BRIDGES

One of the main parameter affecting the simplified collapse estimate is the structural fundamental frequency. Even if an in-situ dynamic characterisation of the structure is suggested, when it is not available a modal elastic analysis or the simplified approach that will be described in this paragraph could be employed.

The modal behaviour of eight bridges with equal span length, piers with different heights (7, 14, 21 m), hinged abutments, continuous elastic deck, with the element geometric dimensions equal to the ones described in the case study chapter, have been analysed and it has been observed that the deck stiffness is substantially affecting the response.

An ad hoc developed index (Z) has been derived to compare the bridge modal behaviour for increasing deck stiffness to a reference response with a deck reduced stiffness.

A modal displacement matrix (U_{10}), with the degrees of freedom (DOFs) displacements of the first three fundamental modes of the bridge with the deck stiffness reduced to the 10% of its elastic value, has been constructed (Eq. 18)

$$\mathbf{U_{10}} = \begin{bmatrix} u_{1,1}^{10} & .. & u_{j,1}^{10} \\ u_{1,2}^{10} & .. & u_{j,2}^{10} \\ u_{1,3}^{10} & .. & u_{j,3}^{10} \end{bmatrix} \tag{18}$$

where U_{10} is the modal eigenvector matrix, $u_{j,i}$ is the modal displacement of the j-th mode at the i-th DOF. The modal displacements have been normalised according to Eq. 19 and a new matrix (Φ_{10}) has been derived.

$$\mathbf{\Phi_{10}} = \frac{\mathbf{U_{10}}}{\sqrt{\mathbf{U_{10}}^{\mathbf{T}}\mathbf{M}\mathbf{U_{10}}}} \tag{19}$$

Then the mode orthogonality with respect to the mass matrix (M) has been verified, observing that the final result of the product in Eq. 20 is the identity matrix (I).

$$\mathbf{I} = \mathbf{\Phi_{10}}^{\mathbf{T}} \times \mathbf{M} \times \mathbf{\Phi_{10}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & .. & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{20}$$

Twelve Φ_k matrixes for each bridge have been constructed with the eigenvectors from the modal linear models where the deck stiffness has been changed from 10% of its elastic value to 300%, normalising the eigenvector with the same procedure described in Eq. 19 for the Φ_{10} case. The response of the structure with different decks has been compared according the difference index (Eq.23) obtained with Eq. 21 and 22:

$$\mathbf{I_N} = \mathbf{\Phi_{10}}^{\mathbf{T}} \times \mathbf{M} \times \mathbf{\Phi_k} \tag{21}$$

$$\mathbf{D} = |\mathbf{I}| - |\mathbf{I_N}| = \begin{bmatrix} D_{11} & .. & D_{1j} \\ .. & .. & .. \\ D_{i1} & .. & Di_j \end{bmatrix} \tag{22}$$

$$Z = \sqrt{\sum D_{ij}^{\ 2}} \tag{23}$$

Fig. 6 shows the relation between the first elastic period and the difference index (Z) described previously for the eight analysed bridges.

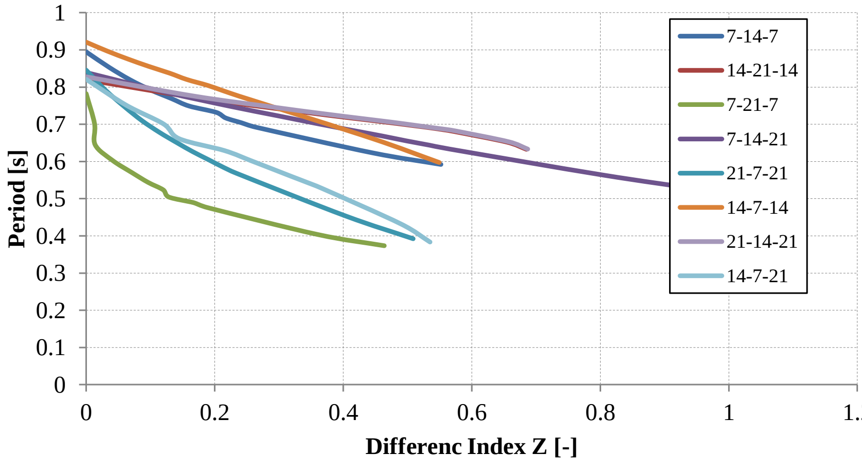

Figure 6. Difference Index for eight different bridges

The difference index is equal to 0 when the deck stiffness is low, while is increasing with increasing flexural and shear deck stiffness. Fig. 6 shows that the slope of all the curve is quite similar and a general expression for the bridge first elastic mode period (T), normalised to the most flexible pier period (T_1) could be extrapolated, as Eq. 24.

$$T = (a + k \cdot Z) \cdot T_1 \tag{24}$$

For the analysed bridges good approximation could be observed with the following values: a = 0.843 and k = -0.479. The value of Z could be chosen between 0 and 1, depending on how the deck is realized (e.g. Z=1 for rigid decks). For safe collapse estimates, its value should be chosen in order obtain the lower PGA from the simplified procedure. The previous expression is an approximation of the observed behaviour and should be employed only when more refined approaches, like an in-situ dynamic characterisation or a modal dynamic analysis, are not available.

CASE STUDY

The simplified method has been applied to a R.C. bridge, with the following pier heights: 7m, 14m and 7m. Fig. 7 shows the pier geometric dimensions and its longitudinal and transversal reinforcements.

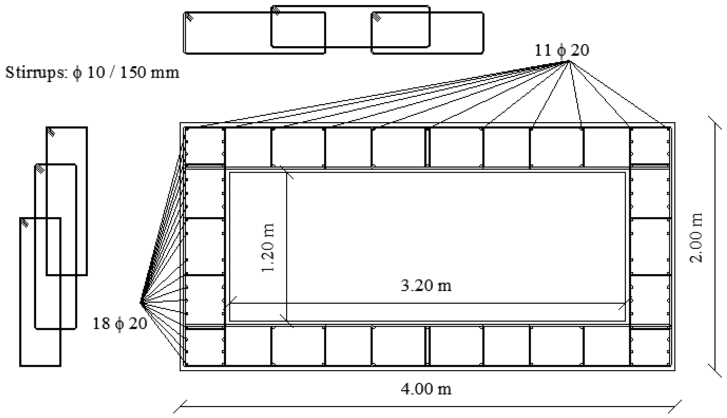

Figure 7. Bridge pier details

Fig. 8 shows the deck dimensions. The reinforced concrete deck is continuous, simply supported by the piers and hinged at the abutments. The material mechanical characteristics have been assumed equal to the ones of a previous experimental tests, see Pinto et al. (2003). The simplified procedure assumes that the failure of the structure will occur due to the collapse of the vertical elements, while the deck will have an elastic behaviour, influencing eventually only the fundamental frequency of the whole structure with its stiffness.

The collapse acceleration from the simplified procedure has been compared with those recorded performing 7 nonlinear time history analyses on the same structure.

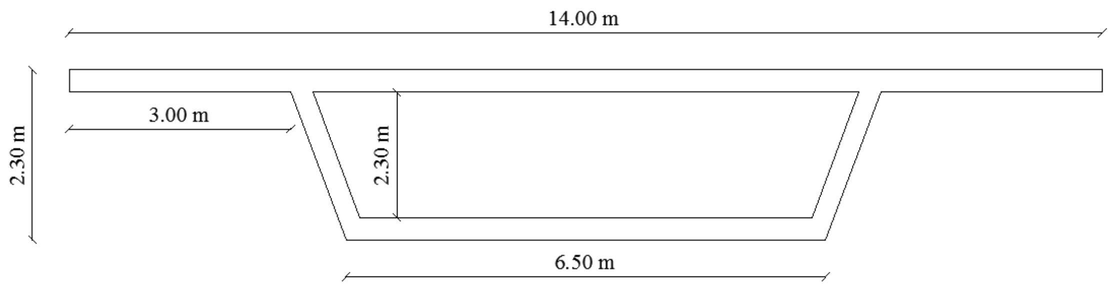

Figure 8. Bridge deck details

The nonlinear model of the piers has been previously validated reproducing the experimental results of pseudo-static tests done on two scaled specimens, described in Peloso (2009). Fig. 9 shows the specimens dimensions. Fig. 10 shows the comparison between the numerical prediction and the experimental response of two different tests performed on two piers: the first one had a 1000 kN axial load while the second a 2000 kN one. A good matching of the numerical prediction to the experimental envelope could be observed.

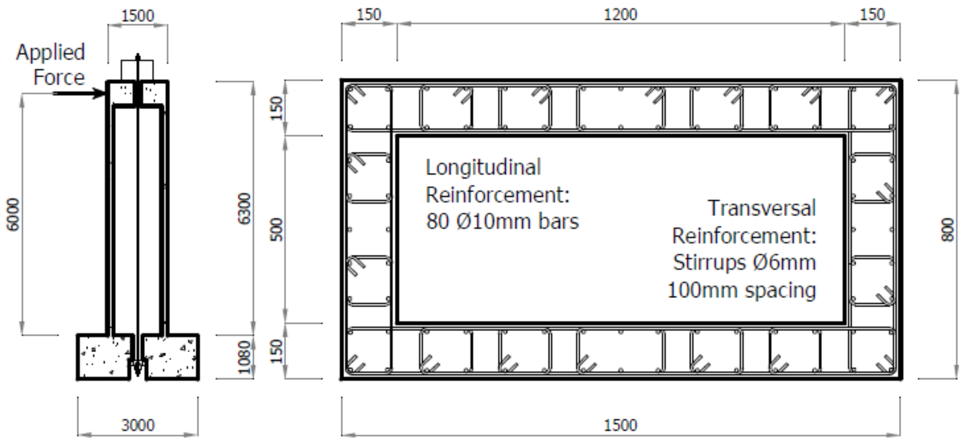

Figure 9. Test set-up adopted for the scaled piers and column details.

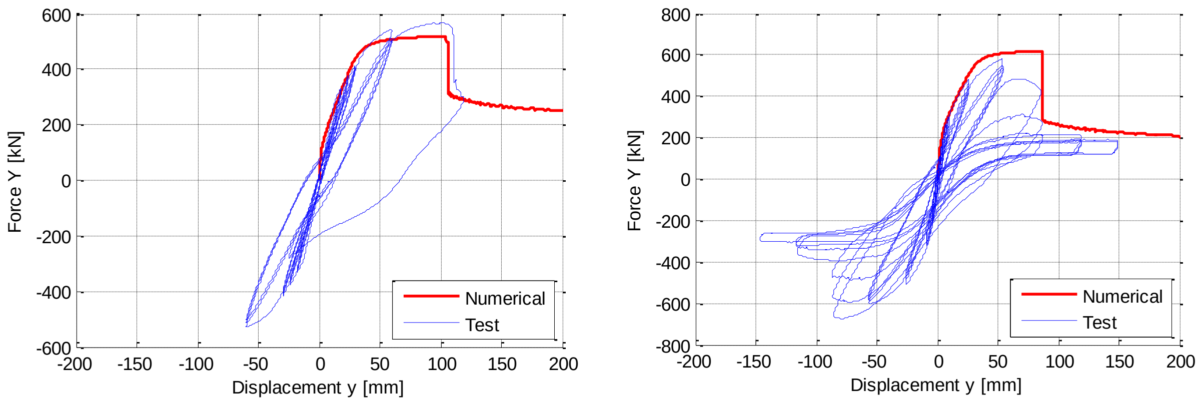

Figure 10. Comparison between the numerical and experimental prediction

The deck mass in the dynamic nonlinear model has been computed with a distributed 20 ton/m load and a distributed 10.6 ton/m load has been modelled to account for the pier mass. A model with fibre displacement based elements, coupled with a linear shear spring, has been implemented to reproduce the nonlinear behaviour of the piers, while the deck has been model with linear elastic elements. As regards the damping, it has been computed with a tangent stiffness Rayleigh approach.

Seven natural accelerograms, with an Italian Code compatible spectrum as shown in Fig. 11, have been selected and applied to the structure. All the accelerograms have been scaled to reach the bridge collapse and the mean peak ground acceleration has been calculated.

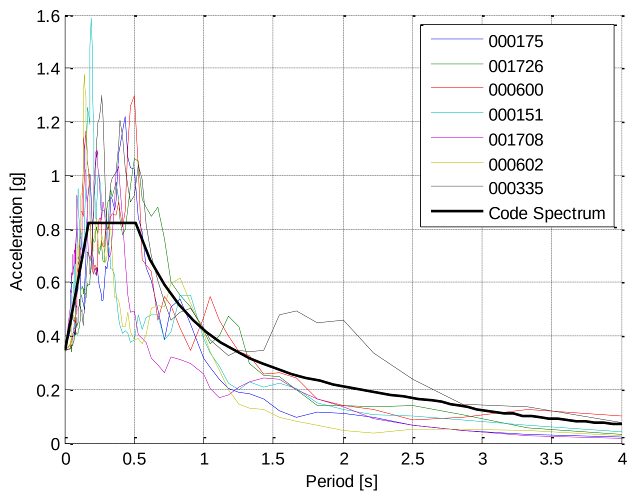

Figure 11. Response spectra of the selected records

Table 1 summarizes the time history results. The mean value of the collapse acceleration from the time history analyses is 0.72g. In all the time history analyses a flexural failure of the 7m piers has been observed.

Table 1. Results of the time history analyses

Record	Collapse acceleration [g]
R151	0.95
R1708	0.78
R1726	0.55
R175	0.70
R335	0.55
R600	0.63
R602	0.85
Mean value	**0.72**

Finally the proposed simplified procedure has been applied to the same structure. The dynamic behaviour has been investigated performing two linear modal analyses: the first with the elastic pier stiffness and the second applying a reduction factor to the pier stiffness in order to have the same cracked stiffness observed performing the elasto-plastic bilinearization of the capacity curve of each pier, as in Fig. 12.

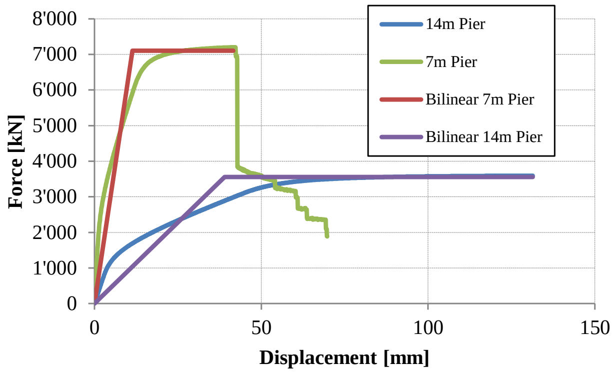

Figure 12. Capacity curves of the selected bridge

The modal response of the elastic structure is more complex, as at least two modes are needed to reach a 80% participating mass of the total one. Table 2 summarizes the modal results of the bridge in the more flexible direction:

Table 2. Modal response of the analysed bridge

Mode	Period [s]	M [%]	Δ_{7m} [-]	Δ_{14m} [-]
1st uncracked	0.31	0.58	0.00524	0.02972
2st uncracked	0.16	0.42	0.02138	-0.00728
1st cracked	0.45	0.82	0.42389	1.05075

The modal mass in the simplified procedure has been computed evaluating only the pertinence mass of each pier, as shown in Fig. 5, and is equal to 3148 ton.
The collapse acceleration has been estimated with two different displacement profiles, as shown in Fig. 13: the simplified one described by Eq. 14 (profile n°1, Table 3) and another one where the displacement of each pier has been computer as the absolute sum of the modal contribution, as suggested by Salsala (2013), (profile n°2, Table 3).

Table 3. Horizontal displacement profiles

Profile	Δ_{7m}	Δ_{14m}	α	$\Delta_{7m,final}$	$\Delta_{14m,final}$	μ_{7m}	μ_{14m}
1	7	14	5.75	40.3	80.5	3.5	2.1
2	1.205	2.025	33.19	40.0	67.2	3.5	1.7

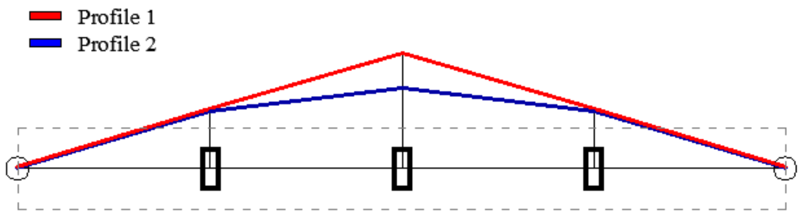

Figure 13. Horizontal displacement profiles

The spectral amplification factor (α_{AD}) has been evaluated both with the first elastic and cracked period in the weaker direction on the Italian Code elastic response spectrum for L'Aquila city, C soil type and T1 slope category. For the specific case study the spectral amplification factor is the same in the two cases, equal to 2.36, as both the elastic and cracked period are laying on the plateau of the elastic spectrum.
The results of the simplified procedure are shown in Table 4. It could be notice a good matching between the simplified estimate and the mean one from the dynamic nonlinear analyses for this case study, with a closer prediction if the second displacement profile is adopted.

Table 4. Simplified procedure estimates

Profile	q_{7m}	q_{14m}	V_{TOT}	PGA [g]
1	3.53	2.06	57433.7	0.79
2	3.50	1.72	55919.2	0.77

CONCLUSIONS

The assessment of the seismic vulnerability of the existing structures is one of the main topics in the Mediterranean countries, particularly in Italy where the great majority of structures were not designed for seismic loads. Fast assessment methods are therefore necessary to evaluate the structural safety and to create priorities between structures which need retrofit interventions or to evaluate the structural safety of damaged structures in post-earthquake scenario.
Many methods have been developed for R.C. buildings, but the research is limited on the development of methods for bridges. It is difficult to implement a general procedure which could be always employed, as there are different construction technologies and the choice of one of them is strongly affecting the response of the global structure. Particularly, it has been observed that the deck stiffness is directly related to the fundamental frequency of the bridge, which is one of the key parameter for the simplified collapse estimate.
A simplified relation, deck stiffness dependent, has been derived and could be employed to estimate the first structural period in the weaker direction when more refined approaches, like in-situ dynamic characterisation or modal dynamic analysis, are not available.
A simplified procedure, analysing only the vertical elements and not the whole structure, has been proposed and applied to a case study structure: a reinforced concrete bridge, with 50 m equal span, continuous reinforced concrete deck simply supported by the piers and hinged at the abutment.
The result of the simplified procedure has been compared with the mean collapse ground acceleration evaluated with a set of 7 time history analyses, performed with an Italian Code compatible response spectrum, observing a good matching of the collapse estimates and the local and global failure mechanism.

Research is still going on aiming to extend the application field of the proposed procedure. In its future releases a simplified way to account for the loss of lateral support of the deck due to its sliding on underdesigned bearings will be introduced; moreover bridges with different abutment restraints will be evaluated.

AKNOWLEDGEMENTS

The research leading to these results received founding by the European Community's Seventh Framework Programme (FP7) under grant agreement n° 262330 for the Project NERA (Network of European Research Infrastructures for Earthquake Risk Assessment and Mitigation).
Additional financial support was received from the Italian Ministry for Instruction, University and Research (MIUR) within the framework of the Programma Operativo Nazionale "Ricerca e Competitività 2007-2013" Regioni Convergenza - "Settore: Ambiente e Sicurezza", Project STRIT (Strumenti e Tecnologie per la gestione del Rischio delle Infrastrutture di Trasporto).

REFERENCES

Calvi G.M., Pavese A., Rasulo A. and Bolognini D. (2005), "Experimental and numerical studies on the seismic response of R.C. hollow bridge piers", *Bulletin of Earthquake Engineering 3:3*, 267-297

Clough RW and Penzien J (1993) Dynamics of Structures, 2nd Ed., McGraw-Hill Book Company, New York

Consiglio Superiore Dei Lavori Pubblici (2008), "Norme Tecniche per le Costruzioni", Gazzetta Ufficiale n°29, Italy

Dolce M, Moroni C (2005) "La valutazione della vulnerabilità e del rischio sismico degli edifici pubblici mediante le procedure VC (vulnerabilità C.A.) e V.M. (vulnerabilità muratura)", *Research report of the Department of Structures, Geotechnics, Geology*, University of Basilicata, Italy, 4/2005

European Committee for Standardization (2010), *Eurocode 8: Design of structures for earthquake resistance – Part 1: General rules, seismic action and rules for buildings*, Brussels

European Committee for Standardization (2010), *Eurocode 8: Design of structures for earthquake resistance – Part 2: Bridges*, Brussels

European Committee for Standardization (2006), *Eurocode 2: Design of concrete structures – Part 1: General rules and rules for buildings.*, Brussels

Pinto A.V. et al. (2003), "Cyclic tests on large-scaled models of existing bridge piers with rectangular hollow cross-section", *Earthquake Engineering and Structural Dynamics*, 32:1995-2012

Peloso S. et al. (2012), "Improvement of a simplified method for the assessment of 3D R.C. frames", *15th World Conference on Earthquake Engineering*, Lisbon, Portugal

Peloso, S., Pavese, A., *"FRP Seismic Retrofit for Insufficient Lap-Splice: Large Scale Testing of Rectangular Hollow Section Bridge Piers"*, paper n. CD232 Proceedings of COMPDYN 2009 ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering Conference, Island of Rhodes, Greece, 2009

Priestley M.J.N., Calvi G.M. and Kowalsky M.J. (2007) Displacement based Seismic Design of Structures, IUSS Press, Pavia

Priestley M.J.N., Seible F. and Calvi G.M. (1996) Seismic Design and Retrofit of Bridges, IUSS Press, Pavia

Salsala A. (2013), "Effect of High Frequency Modes on Pushover Analysis of Structures", International Journal of Science and Research (IJSR), India.