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PARAMETRIC TIME SERIES MODELLING OF SPATIALLY VARIABLE STRONG GROUND MOTION

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ABSTRACT

This paper presents applications of parametric time series in modelling locally homogeneous and stationary ground motion field. Out of the classes of parametric models, the autoregressive model is used in the framework of systems theory, where the recorded ground motion at a set of closely spaced sensors is modelled as the response of a linear time invariant system to white noise excitation. In this framework, estimates of spectral matrix of the ground motion field are readily obtained in terms of the autoregressive parameters. The spectral matrix is composed of auto- and cross-spectral densities of the analysed ground motion signals. Unlike spectral matrices obtained from periodogram estimates, the spectral matrices obtained from autoregressive models are characterized by smaller bias error and variance and do not require any smoothing operations. Such spectral matrices could be useful in various applications in earthquake engineering. In this work, we present examples of such applications in (i) estimating coherency from recorded strong-motion data, and (ii) estimating apparent wave propagation velocity and direction of arrival of seismic body waves. The examples illustrate the advantages of autoregressive spectral estimates over periodogram-based spectral estimates.

INTRODUCTION

Seismic ground motion can vary significantly over the spatial dimensions of horizontally extended structures. It is well known that amplitudes and, to an extent, frequency content of seismic waves are modified while they propagate through the earth. Reduction in seismic wave amplitudes farther away from the source is well-known, and is referred to as ground motion attenuation. Modelling of attenuation effects in engineering applications is most relevant over relatively large spatial distances, in the order of several kilometres. Even at a local scale, for example within spatial extent of one kilometre or so, seismic ground motion is often variable in amplitude, frequency content, and phase. Such local variation within relatively small areas is termed as spatial variability of ground motion. Such local variation has importance in modelling earthquake action on horizontally extended structures in the sense that their supports are moved asynchronously. In this context, spatially variable ground motion is commonly referred to as differential or asynchronous support ground motions.

Spatial variability of ground motion results from various physical effects related to the seismic source, the wave propagation path, and the local site conditions. The main physical effects are:

1. wave passage effects, which refers to the difference in arrival times of seismic waves at different locations;

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2. incoherence effects, which refers to differences in amplitudes and phases due to multiple reflections and refractions of seismic waves in inhomogeneous medium and due to superimposition of waves radiated from different parts of the source;

3. and local site effects, which refers to the change in amplitude and frequency content of ground motion due to local variation of soil conditions.

When the soil medium is locally uniform, variability in amplitude and frequency content is less significant than variations in phase resulting from wave passage and incoherence effects. In such situations, ground motion may be locally modelled as realizations of random processes with spatially uniform amplitude and frequency content (Der Kiureghian, 1996). Further details of these physical effects can be found in Zerva (2009) and the references therein.

The combined result of these physical effects on horizontally extended structures such as dams, bridges, tunnels, pipelines, transmission lines, etc., is asynchronous motion of their supports, commonly referred to as multiple support excitations. The response of a structure under spatially variable ground motion can be interpreted as a combination of two effects: a dynamic effect and a pseudo-static effect. Inertia forces induced by dynamic support motion are responsible for the former, while pseudo-static response refers to the static response (at each time instant) to differential support motion. Dynamic response under uniform soil conditions is generally lowered by random cancellations caused by incoherence and wave passage effects. However, these effects induce pseudo-static response, and the total structural response may increase or decrease in comparison to response due to uniform ground motion (see Zerva, 2009 and references therein). Differential soil conditions tend to increase the pseudo-static response making consideration of spatial variability more important in structural analysis. Asynchronous motion may also excite higher modes of vibration more than that done by uniform support motion. The interplay between structural properties, local site conditions, and characteristics of ground motion is rather complex. It is therefore possible that spatial variability may result in lower or higher structural response than that due to uniform ground motion. It is, however, generally accepted that consideration of spatially variable ground motion is necessary for reliably estimating seismic response of horizontally extended structures (see, for example, Zerva, 2009).

Models of spatially variable ground motion are often calibrated from strong-motion array data from past earthquakes. This includes calculation/specification of apparent wave propagation velocity, coherence function, and site-dependent power spectral density function of seismic waves. Calculation of these quantities is based on auto- and cross-spectral densities of ground motion recorded at closely-spaced locations. Spectral estimation is therefore a vital operation in studying/modelling spatial variation of ground motion. In earthquake engineering, spectral estimation of ground motion characteristics is mainly based Fourier transform-based periodograms. Because periodogram-based spectral estimates are characterized by high variance, smoothing operations are required to derive reliable estimates of spectral densities and the corresponding coherency functions. The selection of the type and bandwidth of smoothing windows is based on experience and subjective decisions, rather than on well-defined statistical criteria. This implies that different analysts may obtain different spectral estimates from the same data by using differing smoothing windows and bandwidths (Broersen, 2006).

Spectral estimators other than those based on periodograms are available in the literature. Parametric time series models (see for example, Box et al., 2008) are favoured over periodograms in many fields of engineering, such as system identification (Ljung, 1999), fluid mechanics (Pavageau et al., 2004), among many others. Despite widespread use of parametric time series models such as autoregressive (AR), and autoregressive-moving-average (ARMA) models are widely used in several engineering disciplines, windowed periodograms still remain the primary tool in spectral analysis of strong-motion array data (Zerva, 2009).

In this article, we describe the application of AR models in spectral analysis of strong-motion array data. The methods and the results presented are based on Rupakhety and Sigbjörnsson (2012, 2013). AR modelling of strong-motion array data, as described in Rupakhety and Sigbjörnsson (2012) is briefly summarized. The application of spectral estimates based on AR models based in frequency-wavenumber analysis of strong-motion array data is also presented, highlighting the main findings in Rupakhety and Sigbjörnsson (2013).

SPECTRAL ESTIMATION

Strong ground motion measured at an array of closely-spaced sensors from an earthquake can be considered as a realization of a random field. The measurement at each sensor is, in this interpretation, a realization of a random process. In general, ground motion at a station is random vector field, consisting of motion along three mutually orthogonal axes. In studying spatial variability, it is common to consider one of the components of motion at every station of the array, thus reducing the vector field to a scalar field. Such a reduction is often argued for by the observation that a transformation of coordinate system into principal directions will render the three components of motion to be uncorrelated (see, Penzien and Watabe, 1975). The validity of such an argument is questionable in the near-fault region where the principal directions are likely to be variable within the array. In this article, we consider one of the ground motion components. This does not imply that we are implying lack of correlation among different components of motion. In reality, significant correlation exists between the different components (see, for example, Sigbjörnsson et al., 2013). We decide to select a single component of motion for simplicity in demonstration and for ease in comparison of results with those published in the literature. We emphasize, however, that the presented methodology can be easily extended to multi-component motion. In the following, a component of discrete (time sampled) ground motion component at a station i is denoted as $a_i\left[k\right]$, with $k=0,1,2,...,N-1$ denoting indices of the signal with N samples. The sampling interval is uniform and is denoted by Δt. It is assumed that the signals have zero mean value.

AR model

In modelling a set of signals recorded by an array, the signals are treated as the response of a linear time invariant (LTI) system fed by a white noise. In this representation, the time invariant system transfer function and the variances of the output noise allows a complete description of the spectral martix of the array signal. The LTI system can be modelled by parametric time series models, such as autoregressive (AR), moving average (MA) and autoregressive-moving-average (ARMA) models. In this study we consider the AR model. The AR model of the signal (ground motion) at a station can be expressed as

$$a_i\left[k\right]=\sum_{r=1}^{p}\alpha_i\left[r\right]a_i\left[k-r\right]+\varepsilon_i\left[k\right] \tag{1}$$

where $a_i\left[k\right]$ is the current value of the sampled signal, $\alpha_i\left[r\right]$ are the model parameters, p is the model order, and $\varepsilon_i\left[k\right]$ represents one-step prediction error which is considered to be sampled from a uncorrelated white noise process with zero mean and variance σ_i^2. The model parameters can be estimated by well-defined statistical techniques such as the least squares method, Yule-Walker equations (Yule, 1927; Walker, 1931), or the Burg method (Burg, 1968). Well-defined statistical criteria exist to determine the optimal model order. The commonly used criteria are the Akaike's final prediction error (FPE) criterion (Akaike, 1970), Akaike's information criteria (AIC) (Akaike, 1974), and Parzen's criterion of autoregressive transfer functions (CAT) (Parzen, 1974). The AIC criterion (see Rupakhety and Sigbjörnsson, 2012) minimizes the value of AIC for a selected model order. A discussion on different estimation methods and model order selection criteria is given in Beamish and Priestley (1981). An estimate of the auto-spectral density of the signal is obtained from the model parameters by using the following equation

$$\bar{S}_{ii}(f)=\frac{\sigma_i^2\Delta t}{\left|1-\sum_{r=1}^{p}\alpha_i[r]\exp(-i2\pi rf\,\Delta t)\right|^2},\quad |f|\le\frac{1}{2\Delta t} \tag{2}$$

where f is frequency measured in Hz, and i is the imaginary unit. This representation is suitable to model the signal at an individual sensor of an array. The signals across the array can be modelled by using multivariate AR models. Considering signals at m sensors of an array with an equal number of samples, the AR model becomes

$$\mathbf{a}[k]=\sum_{r=1}^{p}\boldsymbol{\alpha}[r]\,\mathbf{a}[k-r]+\boldsymbol{\varepsilon}[k] \tag{3}$$

where $\mathbf{a}[k]$ is a vector of the current value of the signals at all the sensors; $\boldsymbol{\alpha}[r]$ is a $m\times m$ matrix of model parameters for each value of $r\in\{1, 2, ..., p\}$; and $\boldsymbol{\varepsilon}[k]$ is a vector of m jointly Gaussian zero- mean uncorrelated white noise processes with variances $\sigma_i^2,\ i\in\{1,2,...,m\}$. The optimal model order is selected by minimizing the AIC for a multivariable model (see, Rupakhety and Sigbjörnsson, 2012). The transfer function matrix of the corresponding LTI system is given by

$$\mathbf{H}(f)=\left[\mathbf{I}-\sum_{r=1}^{p}\boldsymbol{\alpha}[r]\exp(-i2\pi rf\,\Delta t)\right]^{-1} \tag{4}$$

where $\mathbf{I}$ is an identity matrix. The spectral matrix of the signals is then obtained as

$$\bar{\mathbf{S}}(f)=\mathbf{H}(f)\mathbf{C}\mathbf{H}^*(f)\Delta t \tag{5}$$

where the asterisk * denotes conjugate transpose of a matrix; and $\mathbf{C}$ is the covariance matrix of the noise processes, which is a diagonal matrix having σ_i^2 along the diagonal. The spectral matrix contains the auto (power) spectral densities $\left(S_{ii}(f)\right)$ of signals on the main diagonal and the off-diagonal terms $\left(\bar{S}_{ij}(f)\right)$ are the cross-spectral densities (CSD) between the signals $a_i[k]$ and $a_j[k]$. The spectral matrix is thus obtained directly from the model parameters, and no smoothing operations are involved. The spectral estimates describe all the signals of the array optimally in the least squares sense. Furthermore, both model calibration and order determination are based on well-defined statistical criteria. In this sense, the presented spectral estimation is more objective than the periodogram estimates which require subjective judgements in selecting smoothing windows and their bandwidths.

An AR model, as in Equation (3), is then fitted to the signals. Model fitting is based on least squares minimization (see Beamish and Priestley, 1981 for details). Different model orders were tried, and based on the AIC criterion, AR model of order 6 was found to be optimal. Further justification of the selected model order can be found in Rupakhety and Sigbjörnsson (2012).

APPLICATIONS

The spectral estimates derived above find their use in various engineering applications, such as in modelling spatially variable ground motion, in simulating spatially variable ground motion, etc. In this work, two such applications are described based on Rupakhety and Sigbjörnsson (2012, 2013).

Coherency

The coherency between signals a_j and a_k can be obtained as

$$\gamma_{jk}\left(f\right)=\frac{\overline{S}_{jk}\left(f\right)}{\sqrt{\overline{S}_{jj}\left(f\right)\overline{S}_{kk}\left(f\right)}} \tag{5}$$

in which the spectral estimates from Equation 4 are used. The lagged coherency is obtained as

$$\left|\gamma_{jk}\left(f\right)\right|=\frac{\left|\overline{S}_{jk}\left(f\right)\right|}{\sqrt{\overline{S}_{jj}\left(f\right)\overline{S}_{kk}\left(f\right)}} \tag{6}$$

An example of coherency estimation using strong-motion data from the SMART-1 array, located in north-east Taiwan is presented for illustration. The data being used was recorded during the January 29, 1981 earthquake with a local magnitude of 6.3. The epicentre of the earthquake was about 30 km, and the hypocentral depth was about 25 km. The array consists of strong-motion stations arranged along the circumference of three concentric circles: 12 stations each on the outer two rings, three stations on the inner-most ring, and one station at the centre. The radii of the three circles are 200m, 1 km, and 2 km. The central station is denoted as C00, those on the inner circle are numbered I01-I12, and those on the outer circle are numbered O01-O12. The data from this array has been used by many researchers to study spatially variable ground motion (for example, Hao et al., 1989; Oliveira et al., 1991). The data used in this example are the North-South component of ground acceleration recorded at C00, I03, I06, I09, I012, M03, M06, M09, and M12. The S-wave window of 5.12s (512 samples at 100 samples per second) is used. Considering the time of first sample at station O12 as the origin of time, the selected time window corresponds to 7.00-12.12s. The selected signals are pre-processed by de-trending. In addition, the selected window is tapered with a Tukey window. The taper durations of the window is selected as 15% at both ends of the signal. The signals at the inner and middle stations are aligned based on their cross-covariance functions with respect to the signal at C00 (see, Zerva, 2009 for details on signal alignment). The alignment is meant to remove the wave propagation effect. Some of the results are shown in Figure 1. On the main diagonal, the auto-spectral densities are shown. The stations C00, I06, I12, M03, and M12 are numbered as 1, 2, 3, 4, and 5 respectively. Cross-spectral amplitudes (absolute values of CSD) are shown on the plots above the main diagonal while the lagged coherencies are shown on the plots below the main diagonal. In each plot, results based on AR modelling are shown in black while those obtained from windowed periodograms are shown in blue. Periodogram estimates are smoothed with 11 point Hamming windows.

In the very low frequency region (below 0.5 Hz), the variance of spectral amplitudes obtained from the AR estimates was found to be greater than that at higher frequencies. This indicates that the AR spectral estimates are not reliable at such low frequencies. The spectral shapes of the AR process, as seen from Equation 5, approach a constant value as frequency tends to zero. This is contrary to the commonly accepted spectral shapes of earthquake ground motion, which have acceleration decreasing to zero as the frequency tends to zero. This contradiction could be reduced if ground displacements are used in the analysis procedure, thereby obtaining displacement spectra with a constant offset at zero frequency. However, strong-motion data is mostly available in the form of ground acceleration, and due to various noise present in the data, recovery of ground displacement is not always feasible. Due to these reasons it is recommended that ground velocity obtained by integrating ground acceleration is used in AR modelling. Spectral densities of ground acceleration can then be obtained from the spectral densities of ground velocity by a simple transformation (see, Rupakhety and Sigbjörnsson, 2012). The results obtained by using this approach are shown in Figure 2. The results indicate improved behaviour in the low frequency region, where the spectral shapes are consistent with the generally accepted spectral shapes of earthquake ground motions.

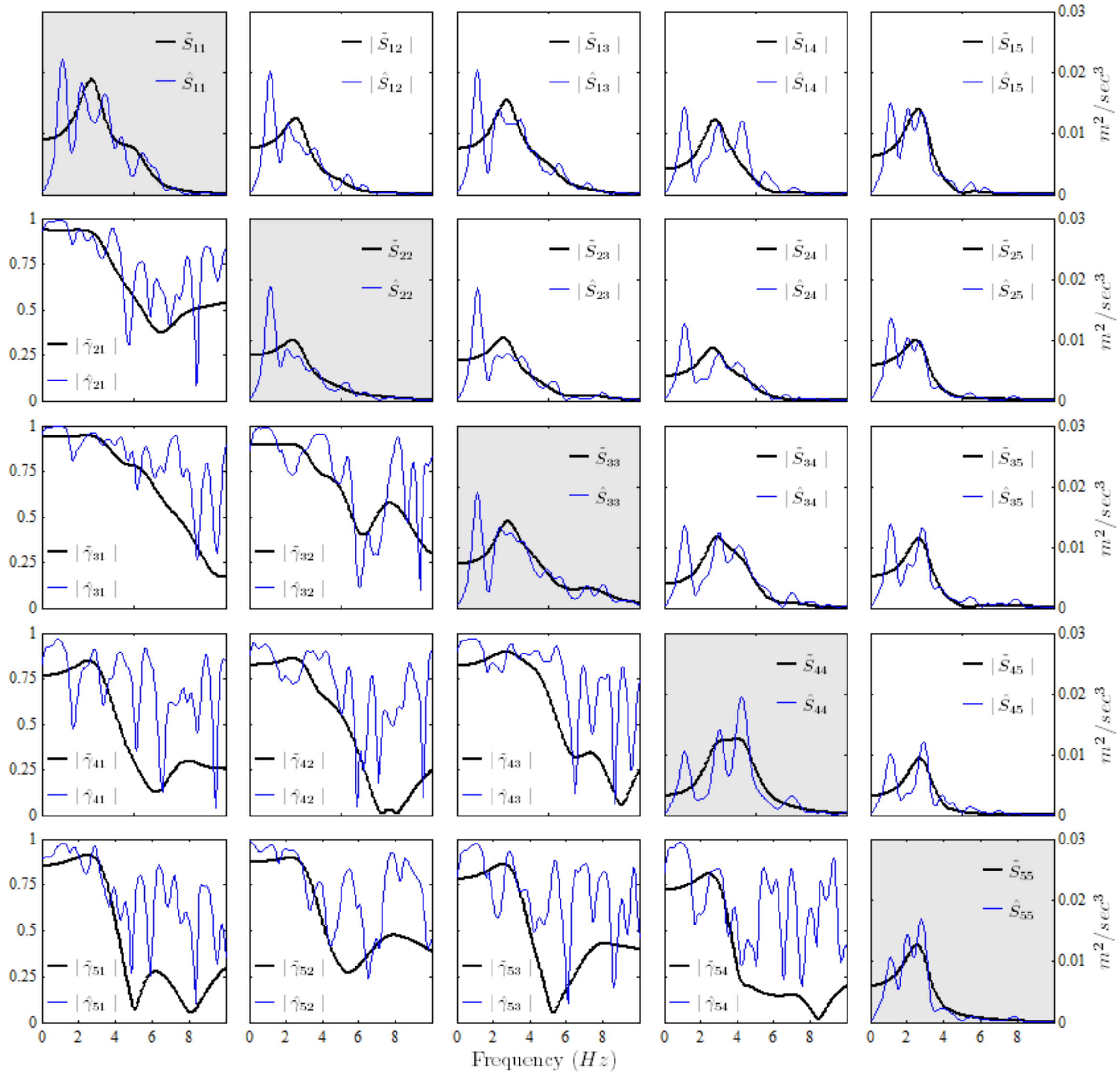

Figure 1. Comparison of the AR and periodogram estimates of spectral densities and lagged coherency between the central, inner ring, and middle ring stations (see text for further descriptions).

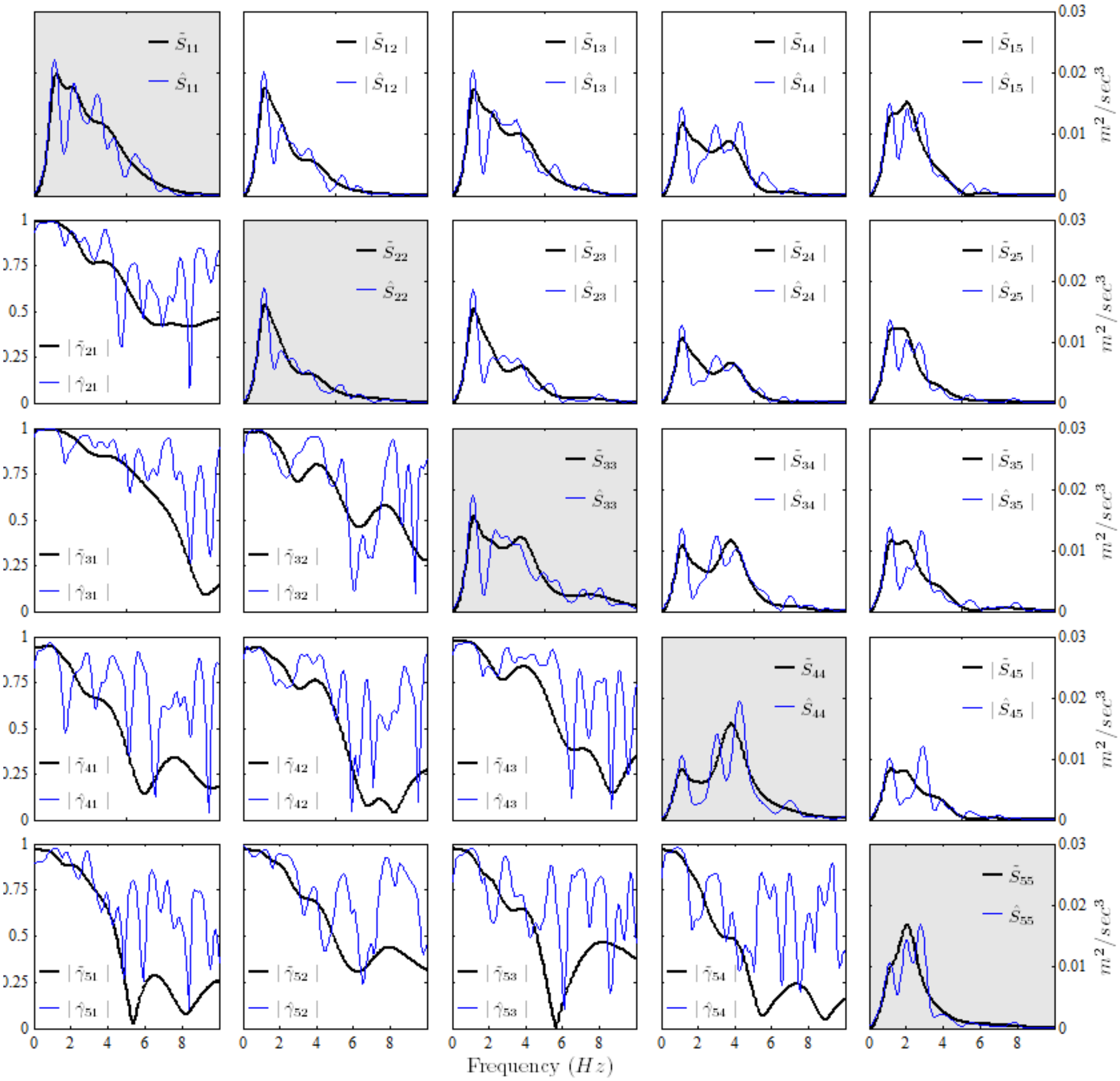

Figure 2. Same as in Figure 1, but the AR estimates are obtained from ground velocity and transformed to spectral densities of ground acceleration.

Frequency-wavenumber (f-k) analysis

Frequency-wavenumber (f-k) spectra are useful to estimate back-azimuth and apparent propagation velocity of impinging waves at arrays. F-k analysis can also be used to understand the contribution of various wave components to the motion detected by array sensors. Several applications of f-k analysis can be found in the earthquake engineering and engineering seismology literature (see Zerva 2009, for a detailed reference list). Back-azimuth (Baz) or direction of arrival (DOA) estimation along with apparent wave propagation velocity can be used to model wave passage effects in studying spatial variability of ground motion. Traditionally f-k spectra are computed from periodogram estimates. Analysis of DOA and wave propagation velocity from such spectra suffers from lack in resolution and spurious spikes due to large variance of periodogram estimates at high frequencies. To overcome this lack of resolution, methods such as multiple signal characterization (MUSIC) are usually employed. In this section, we demonstrate that the AR spectral estimates (Equation 5) provide very good resolution in f-k analysis. For this application, we use the same strong-motion data as was described above. Horizontal components of the central, inner ring and the middle ring stations are used. The spectral computations are based on ground velocity due to the reasons explained above. Using the spectral matrix of Equation 5, the f-k spectra are obtained as

$$F\left(\vec{s}, f\right) = \frac{1}{m^2}\vec{u}^*\left(\vec{s}\right)\bar{\mathbf{S}}(f)\vec{u}\left(\vec{s}\right) \tag{7}$$

where $\vec{s}$ is the slowness vector, $\vec{u}$ represents the beam-steering vector; and m is the number of sensors. The slowness that maximizes the f-k spectra at each frequency gives, for a plane wave, the inverse of apparent wave propagation velocity (c). The direction of the corresponding slowness vector gives the back azimuth (Baz). In this framework, the obtained wave propagation velocities are frequency dependent. However, non-dispersive body waves should have frequency independent slowness. It is common to superimpose the f-k spectra at different frequencies to produce stacked slowness (SS) spectra (see, Spudich and Oppenheimer, 1986). The slowness that maximizes the SS spectra is considered in determining frequency independent wave propagation velocity and back azimuth. The stacked slowness spectra for the data used in this application are shown in Figure 3. For both components of motion on the horizontal plane, the slowness that maximizes the SS spectra is $\{0.1 \quad -0.2\}$ s/km in the East-West and North-South directions, respectively. This results in an apparent propagation velocity of 4.5 km/s and a back azimuth of N153°E.

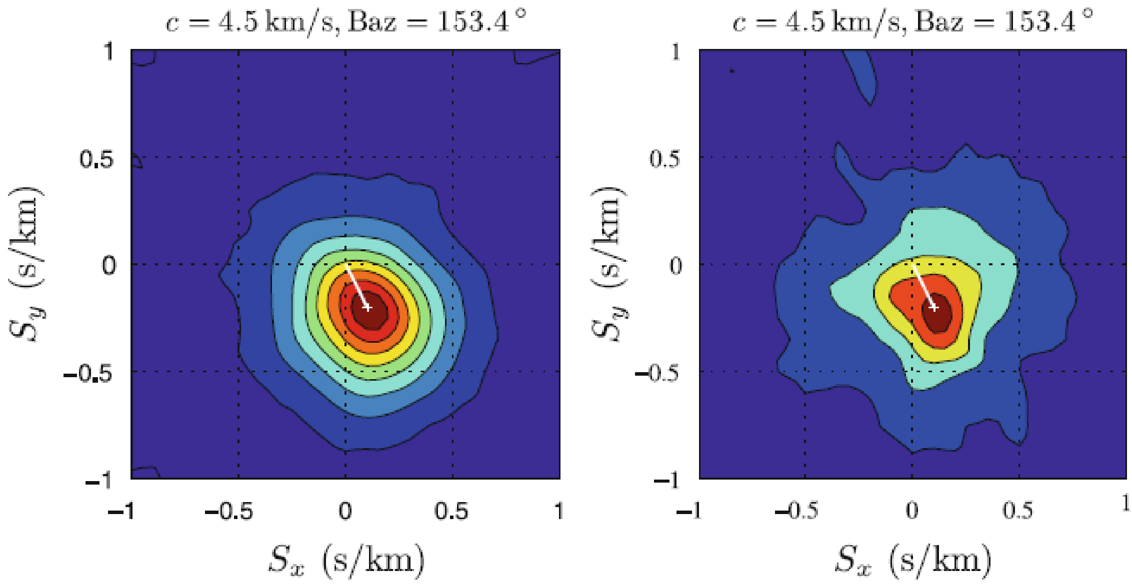

Figure 3. Stacked slowness spectra for North-South and East-West components (left and right panels, respectively) of ground motion. The white cross indicates the peak of the spectra and the white line shows the estimates direction to the source (from Rupakhety and Sigbjörnsson, 2013).

These results presented in Figure 3 are in agreement with those reported in the literature (for example, Zerva, 2009). The estimated back azimuth is in agreement with the source-site geometry. It is noteworthy that the resolution of the SS spectra obtained from AR spectral estimates is higher than that can be obtained from periodogram estimates. The presented results are comparable to the results obtained from MUSIC method, which requires, to a certain degree, subjective decisions from the analyst in separating the signal and noise subspace. Further subjectivity in the MUSIC algorithm is introduced by sub-array spectral averaging (see Bokelmann and Baisch, 1999). It is also noteworthy that the MUSIC-based f-k spectra can only approximately yield the amplitudes of the signals. On the contrary, the AR-based f-k spectra represent both the wavenumber and the amplitude of the analysed signals. In this sense, the AR spectral estimates are superior to other commonly used spectral estimation methods in f-k analysis of strong-motion array data.

CONCLUSIONS

Parametric time series modelling of strong-motion array data has several potential applications in earthquake engineering. A class of parametric models such as the AR, and ARMA models can easily be used in analysing ground motion signals recorded at closely spaced spatial stations. Such modelling allows, within the framework of linear time invariant systems, simultaneous stochastic modelling of multiple signals. Such models are therefore very efficient in modelling locally homogeneous and stationary random fields, such as those of strong ground motion within a small area. While individual and pairwise (two stations at a time) models are commonly employed in computing auto- and cross-spectral densities of strong-motion array data, parametric models are ideally suited for multiple signal processing. In this framework the motion recorded at a set of nearby sensors can be thought of as the response of a linear time invariant system to a white noise excitation. The transfer function of the system along with the noise covariance matrix then provides, an optimal (in the least squares sense), of the spectral matrix describing the random field. As shown in the application presented above, such spectral estimates are superior to traditional periodogram estimates in estimating ground motion coherence, and in direction of arrival analysis. In addition, such spectral matrices are potentially useful in simulating spatially variable ground motion. Within this framework, multi-component motion at every station is easily incorporated, and simulation of three-component ground motion while preserving the correlation structure among different components of recorded ground motion, is feasible. In the spectral representation method, non-stationarity is commonly introduced by applying a time envelope. Simulation using AR models can potentially render such envelopes unnecessary. The sound mathematical framework in estimating and optimizing the parametric models make it more objective than traditional periodogram-based spectral estimates. Further research is needed to explore application and development of parametric time series models in engineering seismology and earthquake engineering applications.

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